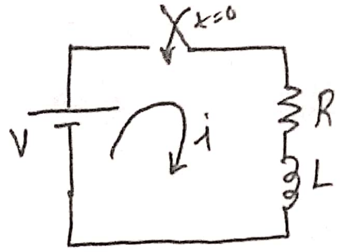
# Transient Analysis of RL and RC Circuits under DC excitations

1) A R-L Series Circuit is suddenly excited by a step input Voltage. Derive the expression for the current as a function of time and draw the graph of current vs time.

Sol:




The series RL circuit shown in figure above has a Constant voltage 'V' applied when the switch is closed.

Applying KVL to mesh, $-V + Ri + L\cdot\frac{di}{dt} = 0$

$$Ri + L\cdot\frac{di}{dt} = V \quad -① $$

Dividing by 'L' and rearranging

$$\frac{R}{L}i + \frac{L}{L}\frac{di}{dt} = \frac{V}{L}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

Taking 'i' as Common $\left(\frac{d}{dt} + \frac{R}{L}\right)i = \frac{V}{L}$

put $\frac{d}{dt} = D \Rightarrow \left(D + \frac{R}{L}\right)i = \frac{V}{L} \quad -②$

Equation ② is a first order, linear differential equation of the type $(D-a)y = Q \quad -③$

The Complete solution of ③, Consisting of the Complementary function and the particular solution, is

$$y = y_c + y_p = ce^{ax} + e^{ax}\int e^{-ax}Q\,dx \quad -④$$

Where c is an arbitrary Constant determined by Known initial Conditions

By ④, the solution of ② with $a = \frac{-R}{L}$, $Q = \frac{V}{L}$

$$i = ce^{-\left(\frac{R}{L}\right)t} + e^{-\left(\frac{R}{L}\right)t} \int e^{\left(\frac{R}{L}\right)t} \left(\frac{V}{L}\right) dt$$

$\int e^{at} dt = \frac{e^{at}}{a}$

$$i = ce^{-\left(\frac{R}{L}\right)t} + e^{-\left(\frac{R}{L}\right)t} \times \frac{e^{\left(\frac{R}{L}\right)t}}{\left(\frac{R}{L}\right)} \times \frac{V}{L}$$

$$i = Ce^{-\left(\frac{R}{L}\right)t} + \frac{L}{R} \times e^{\left(-\frac{R}{L}t + \frac{R}{L}t\right)} \times \frac{V}{L}$$

$$i = Ce^{-\left(\frac{R}{L}\right)t} + \frac{L}{R} \times e^{0} \times \frac{V}{L}$$

$e^{0} = 1$

$$\boxed{i = Ce^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}} \quad - ⑤$$

To determine $c$, we set $t=0$ in ⑤ and substitute the initial current $i_0$ for $i$. Since the current was zero at $t=0^-$, it must also be zero at $t=0^+$. Substituting in ⑤ we obtain

$$i_0 = 0 = c(1) + \frac{V}{R}$$

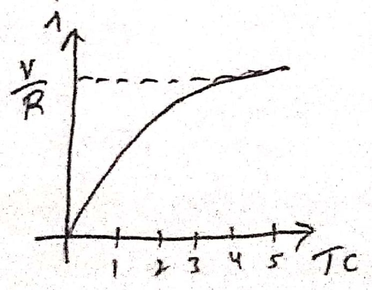
$$c = -\frac{V}{R}$$

Substituting in ⑤

$$i = -\frac{V}{R} e^{-\frac{R}{L}t} + \frac{V}{R}$$

$$\boxed{i = \frac{V}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)}$$

Time Constant (TC) $= \frac{L}{R}$



A series RL Circuit with $R = 50\Omega$ & $L = 10H$ has a Constant Voltage of 100V applied at $t=0$ by closing the switch. Determine i) the equation for $i$, $V_R$ and $V_L$ ii) the Current at $t = 0.5$ secs. iii) the time at which $V_R = V_L$

Sol: The differential equation for the given Circuit is

$$Ri + L.\frac{di}{dt} = V$$

$R = 50\Omega$
$L = 10H$
$V = 100V$

$$50i + 10.\frac{di}{dt} = 10$$

$$\left(D + \frac{R}{L}\right)i = \frac{V}{L} \quad \text{(Refer the derivation) (Memorize)}$$

$$\left(D + \frac{50}{10}\right)i = \frac{100}{10}$$

$$(D+5)i = 10$$

The Complete solution is $i = Ce^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}$ (Memorise)

$$i = Ce^{-5t} + 2$$

At $t = 0^-$, The initial Current is zero.
(before switching)

$$0 = Ce^{0} + 2 \quad \text{(substituting } t=0)$$

$$C = -2.$$

Substituting for c,

1) $$\boxed{i = -2e^{-5t} + 2 = 2\left(1 - e^{-5t}\right) A}$$

$$V_R = iR$$

$$= 2\left(1 - e^{-5t}\right) \times 50$$

$$\boxed{V_R = 100\left(1 - e^{-5t}\right) V}$$

$$V_L = L.\frac{di}{dt} = 10\frac{d}{dt}\left\{-2e^{-5t} + 2\right\}$$

$$V_L = 10 \times -2e^{-5t} \times -5$$

$$\frac{d}{dt}(e^{at}) = ae^{at}$$

$$\boxed{V_L = 100e^{-5t}}$$

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ii) The Current at $t = 0.5$ secs

We have, $i = 2(1-e^{-5t})$

$$i = 2(1-e^{-5\times 0.5})$$

$$i = 2(1-e^{-2.5})$$

$$i = 2(1-0.0082)$$

$$i = 1.9836A$$

iii) When $V_R = V_L$, each must be 50V since the applied voltage is 100V.

$$V_R = 100(1-e^{-5t})$$

$$50 = 100(1-e^{-5t})$$

$$1-e^{-5t} = \frac{5}{100}$$

$$1-e^{-5t} = 0.50$$

$$e^{-5t} = 1-0.5$$

$$e^{-5t} = 0.50$$

Taking $\log_e$ both sides (ln) $\log_e e^{-5t} = \log_e 0.50$

$$-5t \log_e e = \ln 0.50 \qquad \log_e e = 1$$

$$-5t = -0.6931$$

$$t = 0.1386 \text{ sec}$$

OR

$$V_L = 100e^{-5t}$$

$$50 = 100e^{-5t}$$

$$e^{-5t} = 0.50$$

$$\log_e e^{-5t} = \log_e 0.50$$

$$-5t = \ln 0.50$$

$$-5t = -0.6931$$

$$t = 0.1386 \text{ sec}$$

A Constant Voltage is applied to a series RL Circuit by closing the switch. The Voltage across L is 25V at t = 0 and drops to 5V at t = 0.025 sec. If L = 2H, What must be the value of R?

Sol:

$$v_L = V e^{-\frac{R}{L}t}$$ (Memorize)

$L = 2H$, $v_L = 25\,V$ at $t = 0$ $\Rightarrow$ $25 = V e^{-\frac{R}{2}\times 0}$

$$25 = V e^{0}$$

$$V = 25$$

$v_L = 5V$ at $t = 0.025$ sec $\Rightarrow$ $\underset{(V)}{5 = 25}\, e^{-\frac{R}{2}\times 0.025}$

$$e^{-0.0125R} = \frac{5}{25}$$

$$e^{-0.0125R} = 0.2$$

$$\log_e e^{-0.0125R} = \underset{(\ln)}{\log_e 0.2}$$

$$-0.0125R \log_e e = -1.609 \qquad \because \log_e m^n = n\log_e m$$

$$-0.0125R = -1.609$$

$$R = 128.72\,\Omega$$

4) A DC Voltage of 100V is applied to a Coil of resistance $10\Omega$ and inductance 10H. Determine the Value of Current 0.1 sec after switching on and time taken for the Current to reach one-half of its final Value?

Sol:

$$\left(D + \frac{R}{L}\right)i = \frac{V}{L}$$

$R = 10\Omega$
$L = 10H$
$V = 100V$

$$\left(D + \frac{10}{10}\right)i = \frac{100}{10}$$

$$(D+1)i = 10$$

The Complete solution is $i = ce^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}$

$$i = ce^{-1t} + 10$$

At $t=0$, the initial current is zero

$$0 = ce^{0} + 10$$

$$c = -10$$

$$i = -10e^{-1t} + 10 = 10(1-e^{-1t})$$

At $t = 0.1$ sec, $i = 10(1-e^{-0.1})$

$$= 10(1-0.904)$$

$$i = 0.96\ A$$

The final value of current, $i = \frac{V}{R} = 10$

Half of its final value $= 5\ A$

$$i = 10(1-e^{-1t})$$

Time taken to reach half of its final value,

$$5 = 10(1-e^{-t})$$

$$1-e^{-t} = 0.5$$

$$e^{-t} = 0.5$$

$$\log_e e^{-t} = \log_e 0.5$$

$$-t = -0.693$$

$$t = 0.693\ \text{sec.}$$

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A Coil of inductance 10 H and resistance 2 Ω is supplied at 20V. Determine i) Time Constant of the Circuit ii) maximum value of the stored energy.

Sol:

$R = 2\Omega$

$L = 10H$

$V = 20V$

i) Time Constant $= \frac{L}{R} = \frac{10}{2} = 5$ sec

ii) stored energy, $W = \frac{1}{2} L I^2$

$I = \frac{V}{R} = \frac{20}{2} = 10A$

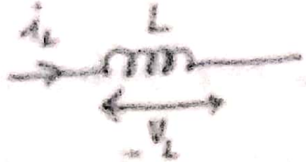
$W = \frac{1}{2} \times 10 \times 10^2 = 500$ joules

**Concept of initial conditions:** In the analysis of networks we write differential equations. We know that the solution of a first order differential equation contains an unknown called as arbitrary constant and in general the number of arbitrary constants present in the solution of a differential equation is equal to the order of the equation. In order to evaluate these arbitrary constants some additional facts about the network are required. This additional information is given in the form of initial conditions. The initial conditions are the conditions that exist in the network at the instant the network equilibrium is altered. The initial conditions are given as the initial values of voltage, current or charge or derivatives of these quantities at the instant of operation of switches, i.e at the instant the network equilibrium is altered. Sometimes we refer to the conditions that exist in the network after sufficiently long time the network equilibrium is altered and they are known as the final conditions.

### 6.2.2 Inductor

The relation between current flowing through inductor & voltage across it is given by,

$$v_L = L\frac{di_L}{dt}$$




If d.c. current flows through inductor, $\frac{di_L}{dt}$ becomes zero as d.c is constant with respect to time. Hence voltage across inductor, $v_L$ becomes zero. Thus, as far as d.c. quantities are considered, in steady state, inductor acts as a short circuit.

We can express inductor current in terms of voltage developed across it as

$$i_L(t) = \frac{1}{L}\int v_L \cdot dt$$

In above equation, the limits of integration are decided by considering past history i.e from $-\infty$ to $t(0^-)$.

Assuming that switching takes place at $t=0$, we can split limits into two intervals as $-\infty$ to $0$ and $0$ to $t$. We have already studied that $0^-$ is the instant just before switching action takes place, while $0^+$ is the instant just after switching action takes place. Hence we can write

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L \cdot dt = \frac{1}{L}\int_{-\infty}^{0^-} v_L \cdot dt + \frac{1}{L}\int_{0^-}^{t} v_L \cdot dt$$

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First term on RHS of equation represents value of inductor current in history period which is nothing but initial condition of $i_L$. Let it be denoted by $i_L(0^-)$

$$\therefore\ i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L \cdot dt$$

At $t=0^+$, we can write,

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L \cdot dt$$

Initially we have assumed that switch acts in zero time. Thus, integration from $0^-$ to $0^+$ is zero.

$$\therefore\ i_L(0^+) = i_L(0^-)$$

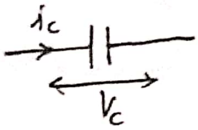
Thus, current through inductor cannot change instantaneously. That means the current through inductor, before and after the switching action is same.

At the time of switching, the voltage across inductor is ideally $\infty$ as time interval $dt$ is zero. Thus, at the time of switching inductor acts as a open circuit. While in steady state at $t=\infty$ it acts as short circuit.

## 6.2.3. Capacitor

The relationship between current through capacitor and voltage across it is given by,

$$i_c = C\frac{dV_c}{dt}$$




If dc voltage is applied to capacitor, $\frac{dV_c}{dt}$ becomes zero as dc voltage is constant with respect to time.

Hence current through capacitor, $i_c$ becomes zero. Thus, as far as dc quantities are considered capacitor acts as a open circuit.

We can express voltage across capacitor in terms of current flowing through it as

$$V_c(t) = \frac{1}{C}\int i_c \cdot dt$$

We can write limits of integration by considering history period.

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c \cdot dt$$

Splitting limits of integration,

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{0^-} i_c \cdot dt + \frac{1}{C}\int_{0^-}^{t} i_c \cdot dt$$

The first term on RHS of above equation represents initial voltage on capacitor. Let it be denoted by $V_c(0^-)$.

At $t = 0^+$, equation is given by

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_c \cdot dt$$

As switch acts in zero time, the integration from $0^-$ to $0^+$ is zero.

$$V_c(0^+) = V_c(0^-)$$

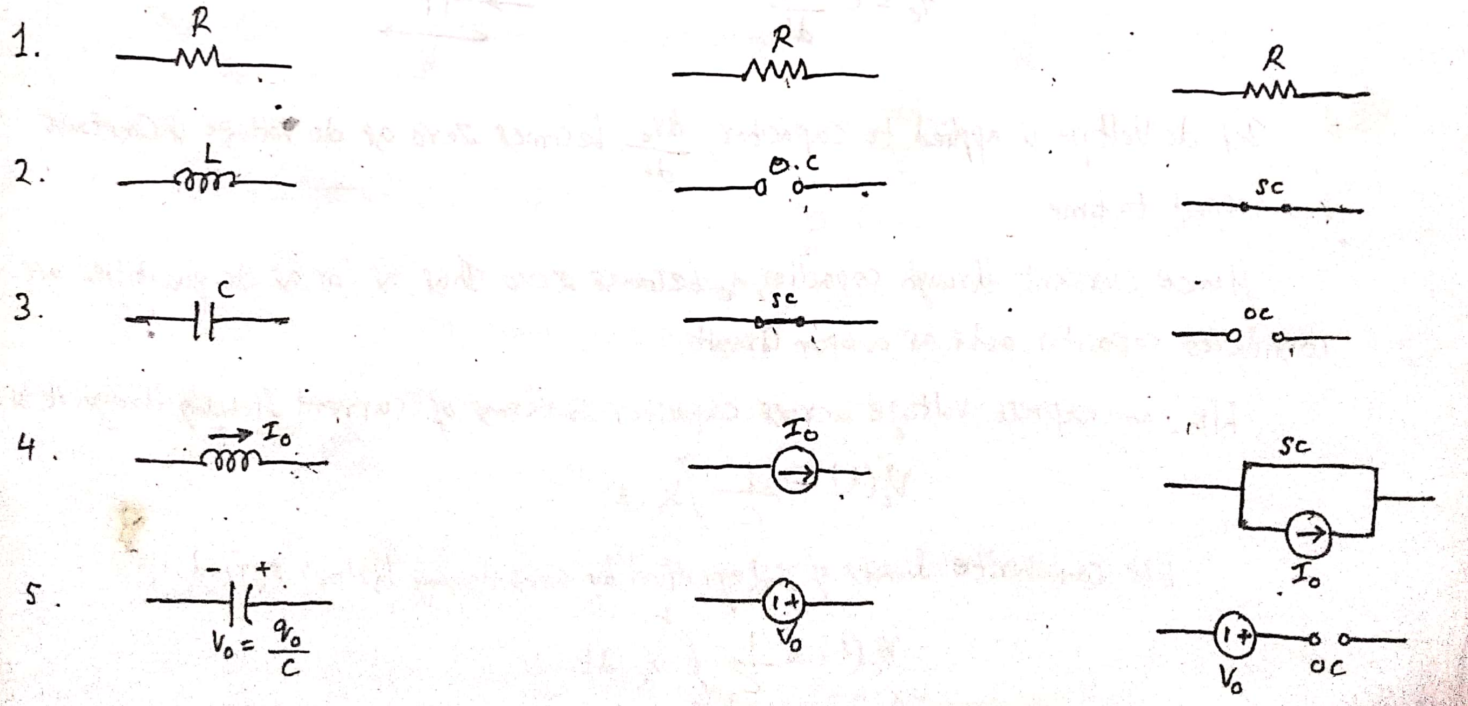
Thus, Voltage across capacitor cannot change instantaneously.

In the network if uncharged capacitor is present then current starts flowing through it instantaneously when excitation is given to the network. So at the instant $t = 0^+$, capacitor acts as a short circuit. once it gets charged at $t = \infty$, in steady state, it acts as an open circuit.

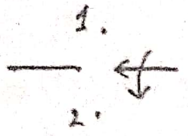
If initially capacitor is charged to voltage $V_0$ before switching, then at instant $t = 0^+$, it acts as a constant voltage source of value $V_0$, while in steady state at $t = \infty$, it acts as open circuit in series with a voltage source.

### 6.2.4 Equivalent form of the elements in terms of the initial & final condition of the element

| | Element | Behaviour immediately after excitation is given at $t=0^+$ instant | Behaviour as $t \to \infty$ i.e steady state |
|---|---|---|---|



**Note :**

i) 

$\Rightarrow$ Switch was in position 1 for a long time, so steady state has reached. The elements are written for $t=\infty$ when the switch was in position 1.

$\Rightarrow$ In position 2, immediately after the operation of switch, the elements are written for $t=0^+$

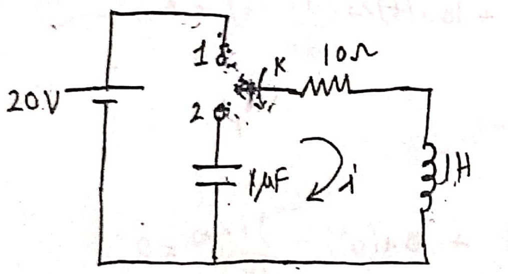
$\Rightarrow$ At $t=0$, switch is moved from 1 to 2. If the arrow mark were opposite, switch is moved from 2 to 1.

ii) At $t=0^+$ & $t=\infty$, the circuit will be purely resistive circuit since inductance and capacitance will be either open or short circuited or with voltage & current sources. So the resistive circuit can be easily solved by making use of mesh analysis or nodal analysis.

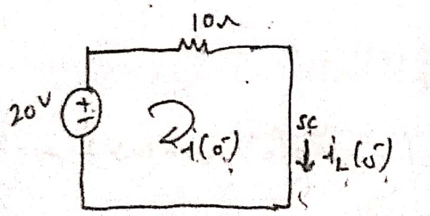
iii)

| Mesh analysis | Time domain equations | Nodal analysis |
|---|---|---|
| $v(t) = R\,i(t)$ | | $i(t) = \dfrac{v(t)}{R}$ |
| $v(t) = L\cdot\dfrac{di(t)}{dt}$ | | $i(t) = \dfrac{1}{L}\displaystyle\int_{-\infty}^{t} v(t)\cdot dt$ |
| $v(t) = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i(t)\cdot dt$ | | $i(t) = C\dfrac{dv(t)}{dt}$ |

**Prob 1.** In the given network of the figure, K is changed from position 1 to 2 at $t=0$. Solve for $i$, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$ if $R=10\Omega$, $L=1H$, $C=1\mu F$, and $V=20V$.




Sol: Circuit for $t=0^-$ : Switch K was in position 1 for a long time, steady state has been reached.

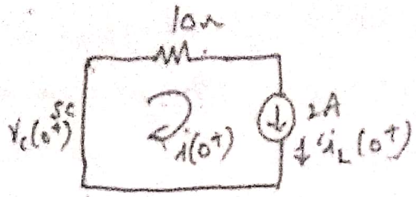



$i(0^-) = \frac{20}{10} = 2A$

$i_L(0^-) = i(0^-) = 2A$

$V_c(0^-) = 0V$ (Since the capacitor is not connected)

Circuit for $t=0^+$ :



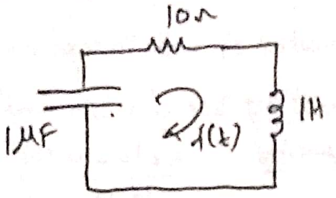

Note: $i_L(0^+) = i_L(0^-)$ Ind current
$V_c(0^+) = V_c(0^-)$ Cap voltage
But $i(0^+) \neq i(0^-)$ Mesh current

$i(0^+) = 2A$

$i_L(0^+) = i_L(0^-) = 2A$

$V_c(0^+) = V_c(0^-) = 0\,V$

Circuit for $t \geq 0^+$




$$\frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i(t).dt + 10\,i(t) + 1.\frac{di(t)}{dt} = 0$$

$$\underbrace{\frac{1}{1\times10^{-6}}\int_{-\infty}^{0^-} i(t).dt}_{V_c(0^-)\ \text{(initial voltage across the capacitor)}} + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t).dt + 10\,i(t) + \frac{di}{dt}(t) = 0$$

$$\frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t).dt + 10\,i(t) + \frac{di}{dt}(t) = 0 \quad -①$$

At $t=0^+$

$$\cancel{\frac{1}{1\times10^{-6}}\int_{0^-}^{0^+} i(t).dt}^{\,0} + 10\,i(0^+) + \frac{di}{dt}(0^+) = 0$$

(switch operates in Zero time)

$$10\,i(0^+) + \frac{di}{dt}(0^+) = 0$$

$$\frac{di}{dt}(0^+) = -10\,i(0^+) = -10\times2$$

$$\frac{di(0^+)}{dt} = -20\ A/sec$$

Differentiating ①

$$\frac{i(t)}{1\times10^{-6}} + 10\,\frac{di}{dt}(0^+) + \frac{d^2 i(t)}{dt^2} = 0$$

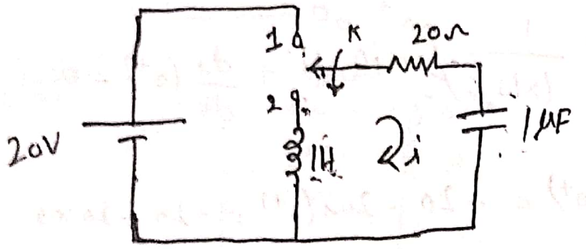
At $t=0^+$

$$\frac{i(0^+)}{1\times10^{-6}} + 10\,\frac{di}{dt}(0^+) + \frac{d^2 i}{dt^2}(0^+) = 0$$

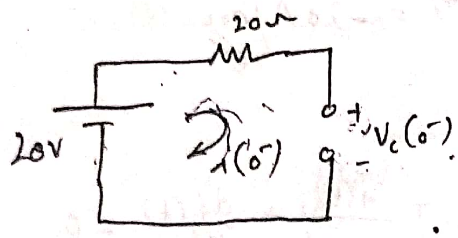
$$\frac{2}{1\times10^{-6}} + 10\times(-20) + \frac{d^2 i}{dt^2}(0^+) = 0$$

$$\frac{d^2 i}{dt^2}(0^+) = -2\times10^{6}\ A/sec^2$$

Prob 2. In the circuit shown in the figure switch K is changed from 1 to 2 at $t=0$. Steady state having been attained in position 1. Find the values of $i$, $\frac{di}{dt}$, $\frac{d^2 i}{dt^2}$ at $t=0^+$.




Sol: Circuit for $t=0^-$

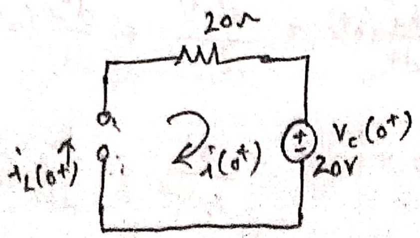



$i(0^-) = 0A$

$V_c(0^-) = 20\,V$

$i_L(0^-) = 0A$ (since the inductor is not connected)

Circuit for $t=0^+$

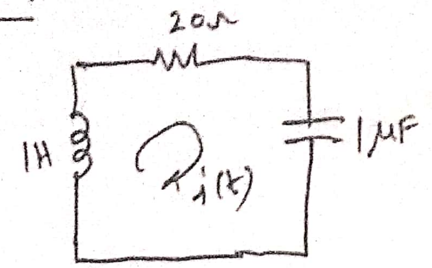



$i(0^+) = 0A$

$i_L(0^+) = 0A$

$V_c(0^+) = V_c(0^-) = 20\,V$

Circuit for $t \geq 0^+$




$$20\,i(t) + \frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i(t).dt + 1.\frac{di}{dt}(t) = 0$$

$$20\,i(t) + \underbrace{\frac{1}{1\times10^{-6}}\int_{-\infty}^{0^-} i(t).dt}_{V_c(0^-)} + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t).dt + 1.\frac{di}{dt}(t) = 0$$

$$20\,i(t) + 20 + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t).dt + 1.\frac{di}{dt}(t) = 0 \quad -(1)$$

At $t = 0^+$

$$20\,i(0^+) + 20 + \frac{1}{1\times10^{-6}}\cancel{\int_{0^-}^{0^+} i(t).dt}^{\,0} + \frac{di}{dt}(0^+) = 0$$

$$\frac{di}{dt}(0^+) = -20 - 20\,i(0^+) = -20 - 20\times 0$$

$$\frac{di}{dt}(0^+) = -20\,A/sec$$

Differentiating (1)

$$20\frac{di}{dt}(t) + \frac{i(t)}{1\times10^{-6}} + \frac{d^2i}{dt^2}(t) = 0$$

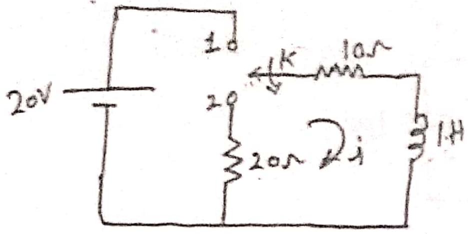
At $t = 0^+$

$$20\frac{di}{dt}(0^+) + \frac{i(0^+)}{1\times10^{-6}} + \frac{d^2i}{dt^2}(0^+) = 0$$

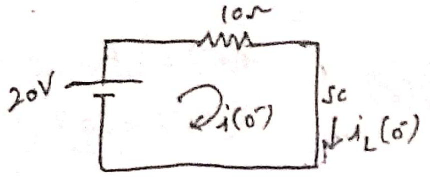
$$20\times(-20) + \frac{0}{1\times10^{-6}} + \frac{d^2i}{dt^2}(0^+) = 0$$

$$\frac{d^2i}{dt^2}(0^+) = +400\,A/sec^2$$

**Prob. 3** In the circuit shown in the figure, switch K is moved from 1 to 2 at $t=0$. Find the values of $i$, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$.



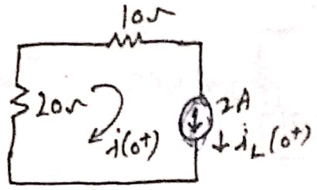

**Sol:** Circuit for $t=0^-$




$i(0^-) = \frac{20}{10} = 2A$

$i_L(0^-) = i(0^-) = 2A$

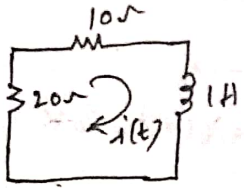
Circuit for $t=0^+$




$i(0^+) = 2A$

$i_L(0^+) = i_L(0^-) = 2A$

Circuit for $t \geq 0^+$




$$30\,i(t) + 1\cdot\frac{di}{dt}(t) = 0 \quad -\ ①$$

At $t=0^+$

$$30\,i(0^+) + \frac{di}{dt}(0^+) = 0$$

$$\frac{di}{dt}(0^+) = -30\,i(0^+) = -30\times 2$$

$$\frac{di}{dt}(0^+) = -60\ A/sec$$

Differentiating ①

$$30\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} = 0$$

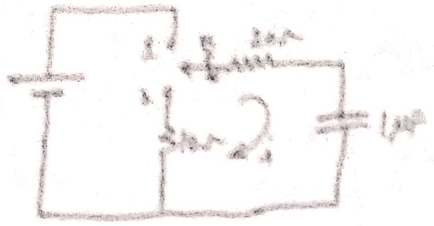
At $t=0^+$

$$30\frac{di}{dt}(0^+) + \frac{d^2i}{dt^2}(0^+) = 0$$

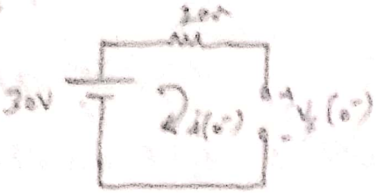
$$\frac{d^2i}{dt^2}(0^+) = -30\times -60$$

$$\frac{d^2i}{dt^2}(0^+) = 1800\ A/sec^2$$

**Prob 4.** In the circuit shown in the figure switch K is changed from 1 to 2 at $t=0$. Find the values of $i$, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$



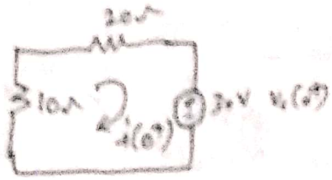

**Sol:** Circuit for $t=0^-$:




$i(0^-) = 0\,A$

$V_c(0^-) = 30\,V$

Circuit for $t=0^+$:



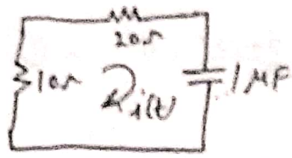

$$+30\,i(0^+) + 30 = 0$$

$$i(0^+) = -1A$$

$$V_c(0^+) = V_c(0^-) = 30V$$

Note: In this problem, mesh current $i(0^+) \neq i(0^-)$

Circuit for $t \geq 0^+$




$$30\,i(t) + \frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i(t)\,dt = 0$$

$$30\,i(t) + \underbrace{\frac{1}{1\times10^{-6}}\int_{-\infty}^{0^-} i(t)\,dt}_{V_c(0^-)} + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t)\,dt = 0$$

$$30\,i(t) + 30 + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t)\,dt = 0 \quad -(1)$$

Differentiating (1)

$$30\frac{di}{dt}(t) + \frac{i(t)}{1\times10^{-6}} = 0 \quad -(2)$$

At $t=0^+$

$$30\frac{di}{dt}(0^+) + \frac{i(0^+)}{1\times10^{-6}} = 0$$

$$\frac{di}{dt}(0^+) = -\frac{1}{30}\,\frac{i(0^+)}{1\times10^{-6}} = -\frac{1}{30}\times\frac{(-1)}{1\times10^{-6}} = 33.333\times10$$

Differentiating ③ $$30\frac{d^2 i(t)}{dt^2} + \frac{1}{1\times10^{-6}}\frac{di(t)}{dt} = 0$$

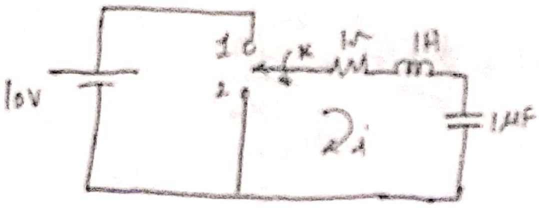
At $t=0^+$

$$30\frac{d^2 i}{dt^2}(0^+) + \frac{1}{1\times10^{-6}}\frac{di}{dt}(0^+) = 0$$

$$\frac{d^2 i}{dt^2}(0^+) = -\frac{1}{30}\times\frac{1}{1\times10^{-6}}\frac{di}{dt}(0^+) = -\frac{1}{30\times10^{-6}}\times 33.333\times10^{3}$$

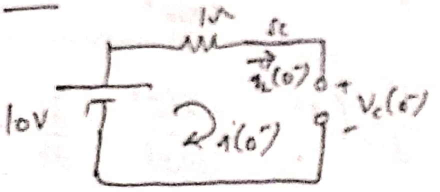
$$\frac{d^2 i}{dt^2}(0^+) = -1.111\times10^{9}\ A/s^2$$

**Prob 5:** In the circuit shown in the figure switch K is changed from 1 to 2 at $t=0$. Find the values of $i$, $\frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ at $t=0^+$.




**Sol:**

Circuit for $t=0^-$

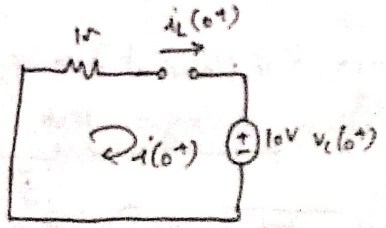



$i(0^-) = 0A$

$i_L(0^-) = 0A$

$V_c(0^-) = 10V$

Circuit for $t=0^+$

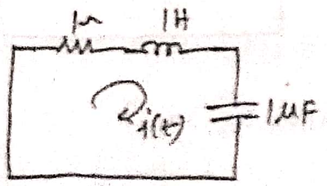



$i(0^+) = 0A$

$i_L(0^+) = i_L(0^-) = 0A$

$V_c(0^+) = V_c(0^-) = 10V$

Circuit for $t \geq 0^+$




$$1.i(t) + 1.\frac{di(t)}{dt} + \frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i(t).dt = 0$$

$$1.i(t) + \frac{di}{dt}(t) + \underbrace{\frac{1}{1\times10^{-6}}\int_{-\infty}^{0^-} i(t)dt}_{V_c(0^-)} + \frac{1}{1\times10^{-6}}\int_{0^+}^{t} i(t)dt = 0$$

$$1\cdot i(t) + \frac{di}{dt}(t) + 10 + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t)\cdot dt = 0 \quad -①$$

At $t=0^+$

$$i(0^+) + \frac{di}{dt}(0^+) + 10 + \frac{1}{1\times10^{-6}}\cancel{\int_{0^-}^{0^+} i(t)\cdot dt}^{\,0} = 0$$

$$\frac{di}{dt}(0^+) = -10 - i(0^+) = -10 - 0$$

$$\frac{di}{dt}(0^+) = -10\ A/sec$$

Differentiating ①

$$\frac{di}{dt}(t) + \frac{d^2i}{dt^2}(t) + \frac{i(t)}{1\times10^{-6}} = 0$$

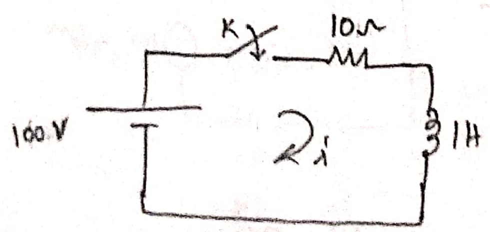
At $t=0^+$

$$\frac{di}{dt}(0^+) + \frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{1\times10^{-6}} = 0$$

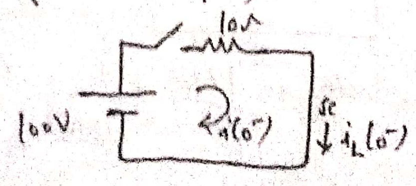
$$\frac{d^2i}{dt^2}(0^+) = \frac{-i(0^+)}{1\times10^{-6}} - \frac{di}{dt}(0^+) = 0 - (-10)$$

$$\frac{d^2i}{dt^2}(0^+) = 10\ A/sec^2$$

**Prob 6:** In the given network, K is closed at $t=0$. Find the values of $i$, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$



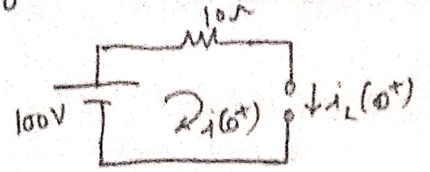

**Sol:** Circuit for $t=0^-$ (switch was open)




$i(0^-) = 0\ A$

$i_L(0^-) = 0\ A$

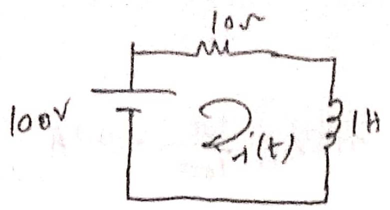
Circuit for $t = 0^+$




$i(0^+) = 0\,A$

$i_L(0^+) = i_L(0^-) = 0\,A$

Circuit for $t \geq 0^+$




$$-100 + 10\,i(t) + 1 \cdot \frac{di}{dt}(t) = 0 \quad - ①$$

At $t = 0^+$

$$-100 + 10\,i(0^+) + \frac{di}{dt}(0^+) = 0$$

$$\frac{di}{dt}(0^+) = 100 - 10\,i(0^+) = 100 - 10 \times 0$$

$$\frac{di}{dt}(0^+) = 100\ A/sec$$

Differentiating ①

$$+10\frac{di}{dt}(t) + \frac{d^2i}{dt^2}(t) = 0$$

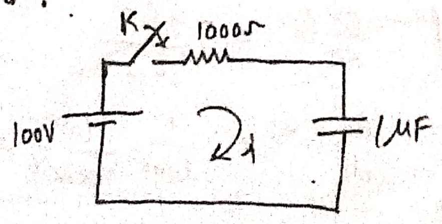
At $t = 0^+$

$$+10\frac{di}{dt}(0^+) + \frac{d^2i}{dt^2}(0^+) = 0$$

$$\frac{d^2i}{dt^2}(0^+) = -10\frac{di}{dt}(0^+) = -10 \times 100$$

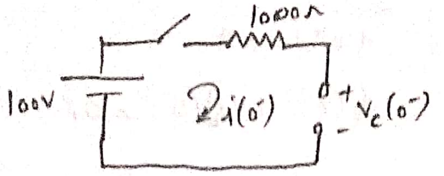
$$\frac{d^2i}{dt^2}(0^+) = -1000\ A/sec^2$$

Prob 7: In the network of the figure, the switch K is closed at $t=0$. Find the values for $i$, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.



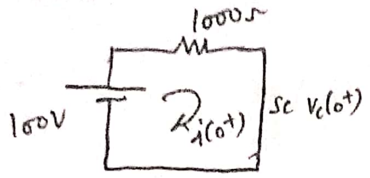
(14)

Sol: Circuit for $t = 0^-$ (Switch was open. capacitor remains uncharged)




$i(0^-) = 0\,A$
$V_c(0^-) = 0\,V$

Circuit for $t = 0^+$

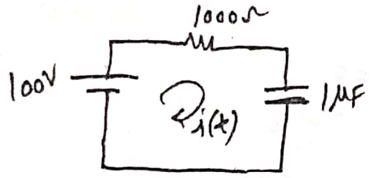



$$i(0^+) = \frac{100}{1000} = 0.1\,A$$

$$V_c(0^+) = V_c(0^-) = 0\,V$$

Note: In this proble[m] $i(0^+) \neq i(0^-)$ (mesh current)

Circuit for $t \geq 0^+$




$$-100 + 1000\,i(t) + \frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i(t)\cdot dt = 0$$

$$-100 + 1000\,i(t) + \underbrace{\frac{1}{1\times10^{-6}}\int_{-\infty}^{0^-} i(t)\cdot dt}_{V_c(0^-)} + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t)\cdot dt = 0$$

$$-100 + 1000\,i(t) + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t)\cdot dt = 0 \quad -①$$

Differentiating ①

$$+1000\,\frac{di}{dt}(t) + \frac{i(t)}{1\times10^{-6}} = 0 \quad -②$$

At $t = 0^+$

$$+1000\,\frac{di}{dt}(0^+) + \frac{i(0^+)}{1\times10^{-6}} = 0$$

$$+1000\,\frac{di}{dt}(0^+) = -\frac{i(0^+)}{1\times10^{-6}}$$

$$\frac{di}{dt}(0^+) = -\frac{1}{1000}\,\frac{i(0^+)}{1\times10^{-6}} = -\frac{0.1}{1000\times1\times10^{-6}}$$

$$\frac{di}{dt}(0^+) = -100\,A/sec$$

Differentiating ②

$$+1000\,\frac{d^2 i(t)}{dt^2} + \frac{1}{1\times10^{-6}}\,\frac{di(t)}{dt} = 0$$

At $t=0^+$

$$+1000\,\frac{d^2 i}{dt^2}(0^+) + \frac{1}{1\times10^{-6}}\,\frac{di}{dt}(0^+) = 0$$

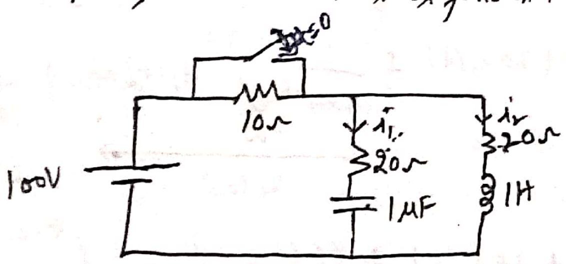
$$+1000\,\frac{d^2 i}{dt^2}(0^+) = \frac{-1}{1\times10^{-6}}\,\frac{di}{dt}(0^+)$$

$$\frac{d^2 i}{dt^2}(0^+) = \frac{1}{1000} \times \frac{-1}{1\times10^{-6}} \times -100$$

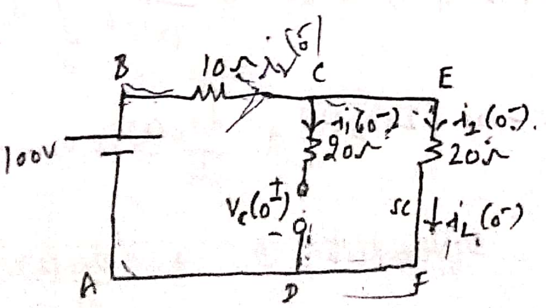
$$\frac{d^2 i}{dt^2}(0^+) = 100\times10^{3}\ A/sec^2$$

Prob 8: A series R-C branch with R = 20Ω and C = 1μF is shunted by an inductor of resistance 20Ω and inductance 1H. This is supplied by a DC source of 100V through a series resistance of 10Ω. There is a switch across 10Ω which is closed at t=0. Solve for the currents in L & C and their derivatives at $t=0^+$.

Sol: For the given description, the network is as follows:




Circuit for $t=0^-$




$i_1(0^-) = 0\,A$

$i_2(0^-) = \frac{100}{10+20} = 3.33\,A$

$i_L(0^-) = i_2(0^-) = 3.33\,A$

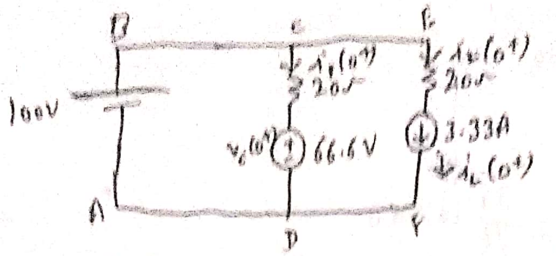
Applying KVL to DCEF: (To find $V_c(0^-)$)

$-V_c(0^-) - 20 i_1(0^-) + 20 i_2(0^-) = 0$

$-V_c(0^-) - 20\times0 + 20\times3.33 = 0$

$V_c(0^-) = 66.6\,V$

Ckt for $t=0^+$ (Switch is closed. A short comes across $10\Omega$. Therefore $10\Omega$ can be neglected)




$V_c(0^+) = V_c(0^-) = 66.6V$

$i_L(0^+) = i_L(0^-) = 3.33A$

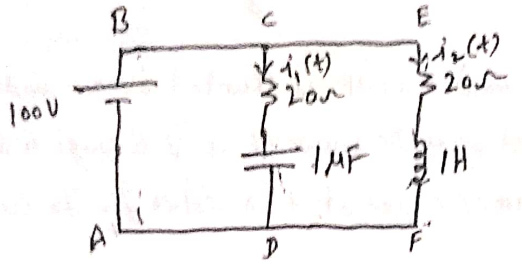
Applying KVL to path ABCD : $-100 + 20i_1(0^+) + 66.6 = 0$

$20i_1(0^+) = 33.4$

$i_1(0^+) = 1.66\,A$

$i_2(0^+) = 3.33A$

Ckt for $t \geq 0^+$




KVL to path ABCD : $-100 + 20i_1(t) + \frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i_1(t)\,dt = 0$

$$-100 + 20i_1(t) + \underbrace{\frac{1}{1\times10^{-6}}\int_{-\infty}^{0^-} i_1(t)\,dt}_{V_c(0^-)} + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i_1(t)\,dt = 0$$

$$-100 + 20i_1(t) + 66.6 + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i_1(t)\,dt = 0 \quad -①$$

KVL to path ABCEFDA : $-100 + 20i_2(t) + 1\cdot\frac{di_2(t)}{dt} = 0 \quad -②$

Differentiating ① $\quad +20\frac{di_1(t)}{dt} + \frac{i_1(t)}{1\times10^{-6}} = 0$

At $t=0^+$

$$+20\times\frac{di_1}{dt}(0^+) + \frac{i_1(0^+)}{1\times10^{-6}} = 0$$

$$\frac{di_1}{dt}(0^+) = -\frac{1}{20}\frac{i_1(0^+)}{1\times10^{-6}} = -\frac{1}{20}\times\frac{1.66}{1\times10^{-6}}$$

$$\frac{di_1}{dt}(0^+) = -83000\ A/sec$$

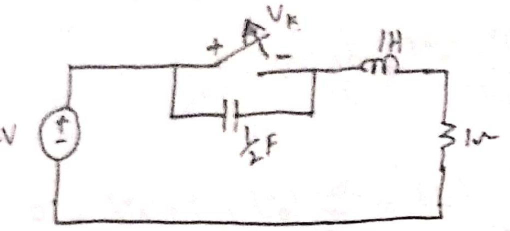
At $t=0^+$, eq ② $\Rightarrow -100 + 20 i_1(0^+) + \frac{di_r}{dt}(0^+) = 0$

$$\frac{di_r}{dt}(0^+) = 100 - 20 i_1(0^+) = 100 - 20\times 3.33$$

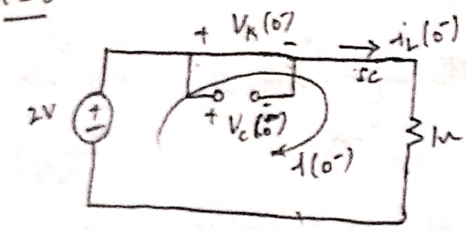
$$\frac{di_r}{dt}(0^+) = 33.33 \text{ A/sec}$$

Prob 9: $R = 1\Omega$, $L = 1H$ and $C = \frac{1}{2}F$ are in series with a switch across capacitor, 2V is applied to the circuit. At $t=0^-$ the switch is in closed position. At $t=0$ the switch is opened. Find at $t=0^+$, the voltage across the switch, its first and second derivatives.

Sol: For the given description, the network is as follows:




Circuit for $t=0^-$

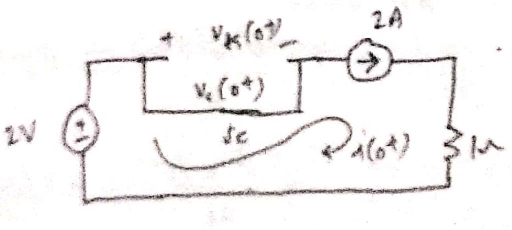



$V_K(0^-) = 0V$

$V_c(0^-) = 0V$

$i(0^-) = \frac{2}{1} = 2A$ $\quad i_L(0^-) = \frac{2}{1} = 2A$

Circuit for $t=0^+$



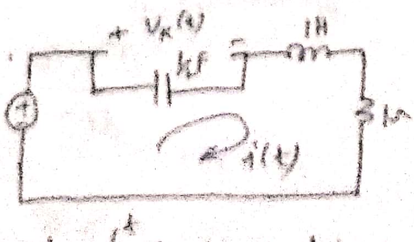

$V_K(0^+) = 0V$

$V_c(0^+) = V_c(0^-) = 0V$

$i_L(0^+) = i_L(0^-) = 2A$

$i(0^+) = 2A$

Circuit for $t \geq 0^+$




$$-2 + \frac{1}{\frac{1}{2}}\int_{-\infty}^{t} i(t)\,dt + 1\cdot\frac{di(t)}{dt} + 1\cdot i(t) = 0$$

$$-2 + \underbrace{2\int_{-\infty}^{0^-} i(t)\,dt}_{V_c(0^-)} + 2\int_{0^-}^{t} i(t)\,dt + 1\cdot\frac{di(t)}{dt} + 1\cdot i(t) = 0$$

$$-2 + 2\int_{0^-}^{t} i(t).dt + 1.\frac{di(t)}{dt} + 1.i(t) = 0 \quad - ①$$

At $t = 0^+$

$$-2 + 2\overbrace{\int_{0^-}^{0^+} i(t).dt}^{0} + 1.\frac{di}{dt}(0^+) + 1 i(0^+) = 0$$

$$\frac{di}{dt}(0^+) = 2 - 1\,i(0^+) = 2 - 1\times 2$$

$$\frac{di}{dt}(0^+) = 0 \text{ A/sec}$$

Differentiating ①

$$+2i(t) + \frac{d^2 i(t)}{dt^2} + \frac{di(t)}{dt} = 0$$

At $t = 0^+$

$$+2i(0^+) + \frac{d^2 i}{dt^2}(0^+) + \frac{di}{dt}(0^+) = 0$$

$$\frac{d^2 i}{dt^2}(0^+) = -\frac{di}{dt}(0^+) - 2i(0^+) = 0 - 2\times 2 = -4 \text{ A/sec}^2$$

Voltage across the switch, $V_K$ = Voltage across capacitor $[V_K(t) = V_c(t)]$

$$V_K(t) = \frac{1}{\frac{1}{2}}\int_{-\infty}^{t} i(t).dt = \underbrace{2\int_{-\infty}^{0^-} i(t).dt}_{V_c(0^-)} + 2\int_{0^-}^{t} i(t).dt$$

$$\boxed{V_K(t) = 2\int_{0^-}^{t} i(t).dt \quad - ②}$$

Differentiating ② $\quad \frac{dV_K(t)}{dt} = 2i(t) \quad - ③$

At $t = 0^+$

$$\frac{dV_K(0^+)}{dt} = 2i(0^+) = 2\times 2$$

$$\frac{dV_K(0^+)}{dt} = 4 \text{ V/sec}$$

Differentiating ③

$$\frac{d^2 V_K(t)}{dt^2} = 2\frac{di(t)}{dt}$$

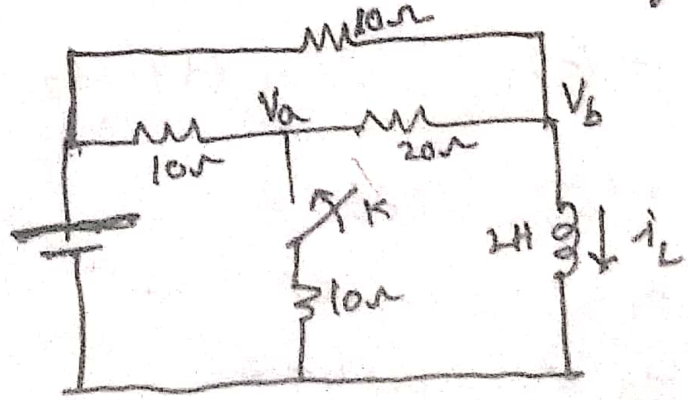
At $t = 0^+$

$$\frac{d^2 V_K(0^+)}{dt^2} = 2\frac{di}{dt}(0^+)$$

$$\frac{d^2 V_K(0^+)}{dt^2} = 2\times 0 = 0 \text{ V/sec}^2$$

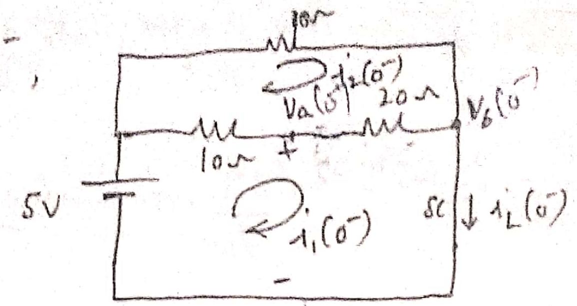
$dt^2$

In the network shown in the figure, a steady state is reached with the switch K open. At $t=0$, the switch is closed. For the element values given, determine the value of $V_a(0^-)$ and $V_a(0^+)$, $V_b(0^-)$, $V_b(0^+)$.



Sol:

At $t = 0^-$,




KVL to mesh 1 : $-5 + 30\{i_1(0^-) - i_2(0^-)\} = 0$

$$30i_1(0^-) - 30i_2(0^-) = 5 \quad -(1)$$

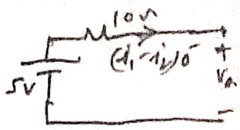
KVL to mesh 2 : $+10i_2(0^-) + 30\{i_2(0^-) - i_1(0^-)\} = 0$

$$-30i_1(0^-) + 40i_2(0^-) = 0 \quad -(2)$$

Solving (1) & (2) $\quad i_1(0^-) = 0.666\text{ A}$

$i_2(0^-) = 0.5\text{ A}$

$V_b(0^-) = 0\text{ V}$ //



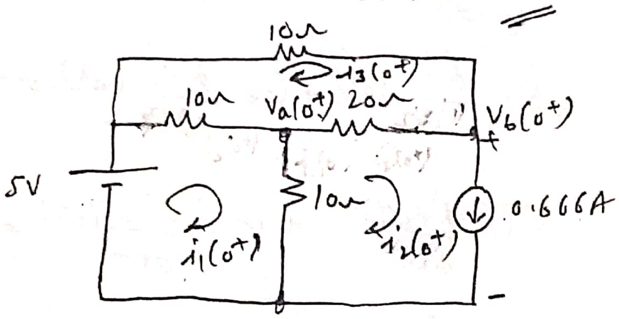

$i_L(0^-) = i_1(0^-) = 0.666\text{ A}$

$V_a(0^-) = 5 - 10\{i_1(0^-) - i_2(0^-)\}$ $\quad -5 + 10(i_1 - i_2) + V_a(0^-) = 0$

$= 5 - 10(0.666 - 0.5)$

$V_a(0^-) = 3.34\text{V}$

At $t = 0^+$,




$i_L(0^+) = i_L(0^-) = 0.666\text{A}$

KCL to non essential mesh : $i_2(0^+) = 0.666\text{ A}$ // $\quad -(1)$

KVL to essential mesh 1 : $-5 + 10(i_1(0^+) - i_3(0^+)) + 10(i_1(0^+) - i_2(0^+))$

KVL to essential mesh 3 : $+10i_3(0^+) + 20(i_3(0^+) - i_2(0^+)) + 10(i_3(0^+) - i_1(0^+)) = 0$

(1) $\Rightarrow (0)i_1(0^+) + (1)i_2(0^+) + (0)i_3(0^+) = 0.666$

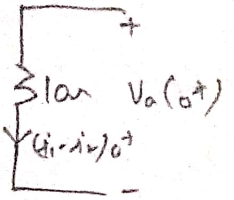
(2) $\Rightarrow (+20)i_1(0^+) + (-10)i_2(0^+) + (-10)i_3(0^+) = +5$

(3) $\Rightarrow (-10)i_1(0^+) + (-20)i_2(0^+) + (+40)i_3(0^+) = 0$

$i_1(0^+) = 0.8565\text{ A}$

$i_2(0^+) = 0.666\text{A}$

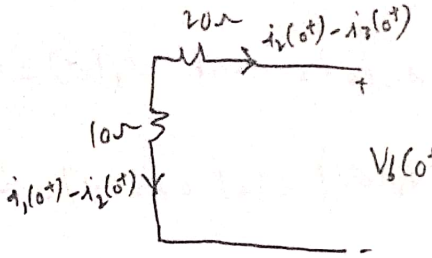
$i_3(0^+) = 0.5471\text{A}$




$$V_a(0^+) = 10\left(i_1(0^+) - i_2(0^+)\right)$$

$$= 10\left(0.8565 - 0.666\right)$$

$$V_a(0^+) = 1.905\,V$$

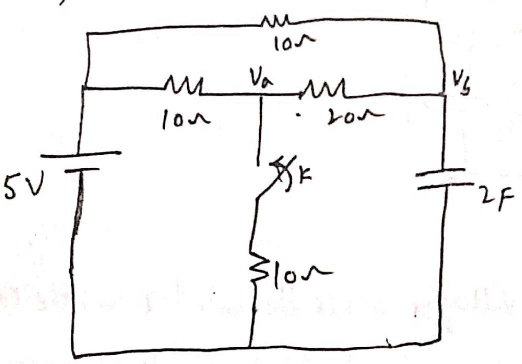



$$-V_b(0^+) + 20\left\{i_2(0^+) - i_3(0^+)\right\} + 10\left\{i_1(0^+) - i_2(0^+)\right\} = 0$$

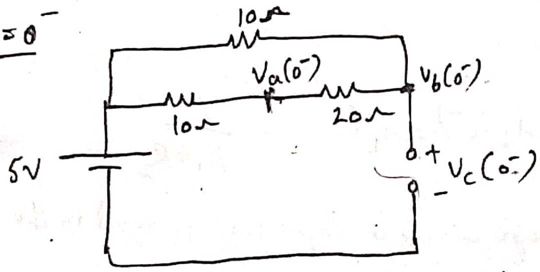
$$V_b(0^+) = -20\left(0.666 - 0.5471\right) + 10\left(0.8565 - 0.666\right)$$

$$V_b(0^+) = -0.473\,V$$

15. In the accompanying figure is shown a network in which steady state is reached with switch K open. At $t=0$, the switch is closed. For the element values given, determine the values of $V_a(0^-)$, $V_a(0^+)$, $V_b(0^-)$ & $V_b(0^+)$.



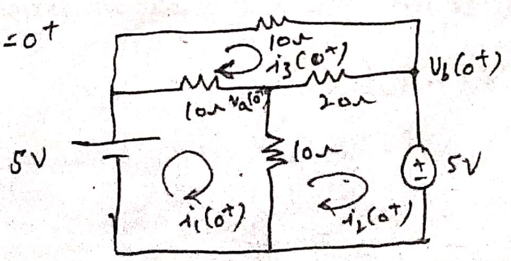

Sol: Circuit for $t=0^-$




Since the source is not connected, current is zero in all branches.

$$V_c(0^-) = 5V, \quad V_a(0^-) = 5V, \quad V_b(0^-) = 5V$$

Circuit for $t=0^+$



The KVL equations are: $(20)\, i_1(0^+) + (-10)\, i_2(0^+) + (-10)\, i_3(0^+) = 5$

(By inspection)

$(-10)\, i_1(0^+) + (30)\, i_2(0^+) + (-20)\, i_3(0^+) = -5$

$(-10)\, i_1(0^+) + (-20)\, i_2(0^+) + (40)\, i_3(0^+) = 0$

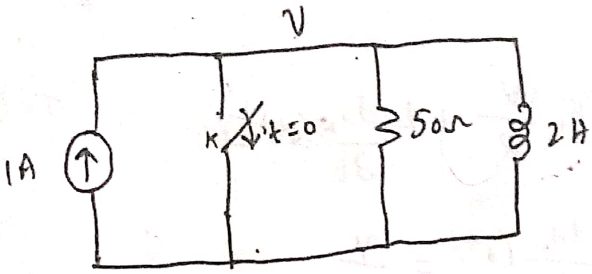
$i_1(0^+) = 0.2\,A \quad i_2(0^+) = -0.1\,A \quad i_3(0^+) = 0\,A$

$V_a(0^+) = 10\{i_1(0^+) - i_2(0^+)\} = 10(0.2 + 0.1) = 3V$

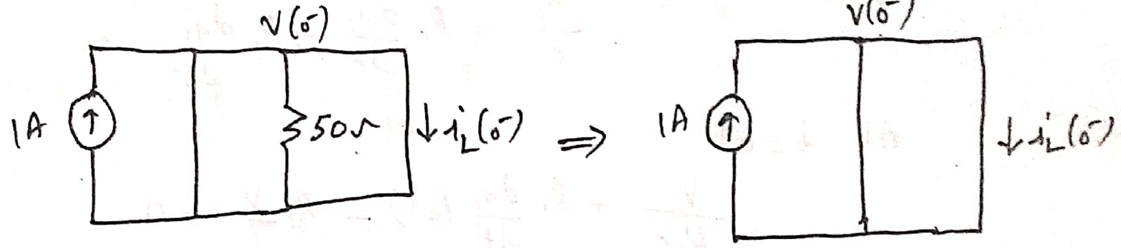
$V_b(0^+) = 5V$

For the circuit shown in the figure, the switch K is opened at $t=0$, find the values of $V$, $\frac{dV}{dt}$, $\frac{d^2V}{dt^2}$ at $t=0^+$

KCL



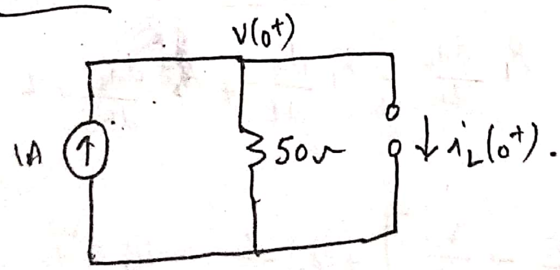

Sol: Circuit for $t=0^-$




$i_L(0^-) = 0A$

$V(0^-) = 0V$

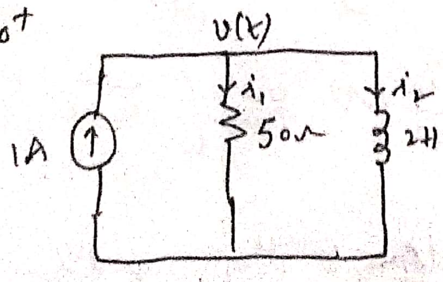
Circuit for $t=0^+$




$i_L(0^+) = i_L(0^-) = 0A$

$V(0^+) = 50 \times 1 = 50V$

Circuit for $t \geq 0^+$




Apply KCL: $i_1 + i_2 = 1$

Nodal equation

$$\frac{v(t)}{50} + \frac{1}{2}\int_{-\infty}^{t} v(t)\,dt = 1$$

$$\frac{v(t)}{50} + \underbrace{\frac{1}{2}\int_{-\infty}^{0^-} v(t)\,dt}_{i_L(0^-)=0} + \frac{1}{2}\int_{0^-}^{t} v(t)\,dt = 1$$

$$\frac{v(t)}{50} + \frac{1}{2}\int_{0^-}^{t} v(t)\,dt = 1 \quad -①$$

Differentiating ①

$$\frac{1}{50}\frac{dv(t)}{dt} + \frac{1}{2}v(t) = 0 \quad -②$$

At $t=0^+$

$$\frac{1}{50}\frac{dv}{dt}(0^+) + \frac{1}{2}v(0^+) = 0$$

$$\frac{dv}{dt}(0^+) = -50 \times \frac{1}{2} \times 50 = -1250\ V/sec$$

Differentiating ②

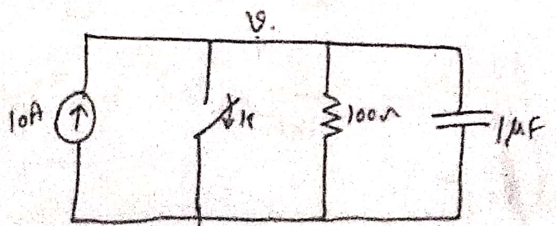
$$\frac{1}{50}\frac{d^2v}{dt^2}(t) + \frac{1}{2}\frac{dv(t)}{dt} = 0$$

At $t=0^+$ $$\frac{d^2v}{dt^2}(0^+) = -50 \times \frac{1}{2}\frac{dv}{dt}(0^+)$$

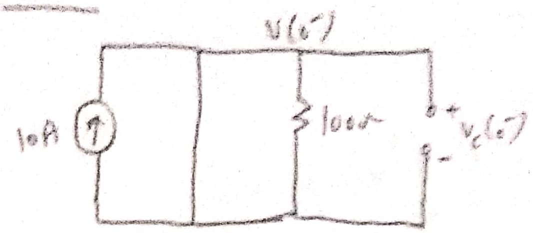
$$= -50 \times \frac{1}{2} \times (-1250)$$

$$\frac{d^2v}{dt^2}(0^+) = 31.25 \times 10^3\ V/sec^2$$

22. In the circuit shown in the figure, switch K is opened at $t=0$, find the values of $v$, $\frac{dv}{dt}$ & $\frac{d^2v}{dt^2}$ at $t=0^+$



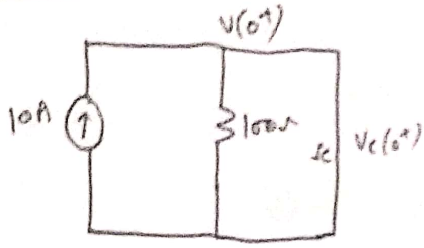
Sol: Circuit $t = 0^-$




$100\Omega$ can be neglected. All the current flows through short

$$V_c(0^-) = 0 \quad , \quad V(0^-) = 0$$

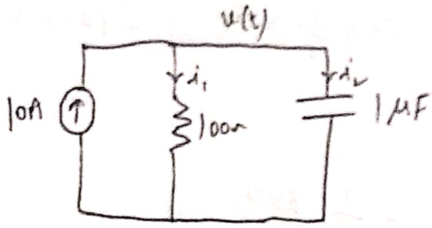
Circuit for $t = 0^+$




$$V_c(0^+) = V_c(0^-) = 0\,V$$

$V(0^+) = 0\,V$ ... (Since no current flows through $100\Omega$)

Circuit for $t \geq 0^+$




$$i_1 + i_2 = 10$$

$$\frac{v(t)}{100} + 1\times10^{-6}\frac{dv(t)}{dt} = 10 \quad -①$$

At $t = 0^+$

$$\frac{v(0^+)}{100} + 1\times10^{-6}\frac{dv}{dt}(0^+) = 10$$

$$\frac{dv}{dt}(0^+) = \frac{+1}{1\times10^{-6}}\left[\frac{-\overset{\to 0}{v(0^+)}}{100} + 10\right]$$

$$\frac{dv}{dt}(0^+) = +\frac{1}{1\times10^{-6}}(10) = 10^7\ \text{V/sec}$$

Differentiating ① and at $t = 0^+$

$$\frac{1}{100}\frac{dv(0^+)}{dt} + 1\times10^{-6}\frac{d^2v}{dt^2}(t) = 0 \Rightarrow \frac{d^2v}{dt^2}(0^+) = -10^{11}\ \text{V/sec}^2$$